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| **Unit II** |
| **Random Variable**  A random variable is a function that assigns a real number to every element of sample space. Let S be the sample space of an experiment. Here we assign a specific number to each outcome of the sample space. A random variable X is a function from S to the set of real numbers R i.e. X: S →R  **Ex.** Suppose a coin is tossed twice S = {HH, HT, TH, TT}.Let **X:** represents number of heads on top face. So to each sample point we can associate a number X (HH) = 2, X (HT) = 1, X (TH) = 1, X (TT) = 0.Thus X is a random variable with range space RX = { 0, 1, 2}  **Types of random Variable:**  **Discrete Random Variable:** A random variable which takes finite number of values or countable infinite number of values is called discrete random variable.  Example: Number of alpha particles emitted by a radioactive source.  **Continuous Random Variable:** A random variable which takes non-countable infinite number of values is called discrete random variable. Example: length of time during which a vacuum tube is installed in a circuit functions is a continuous RV.  **Discrete Probability Distribution**  Suppose a discrete variate X is the outcome of some experiment. If the probability that X takes the value xi is pi, then    Where    The set of values xi with their probabilities pi i.e. (xi,pi) constitute a discrete probability distribution of the discrete variate X. The function p is called probability mass function (pmf) or probability density function (pdf).  **Cumulative Distribution Function (CDF) or Distribution Function** of discrete random variable X is defined by F(x) = p(X ≤ x) where x is a real number (– ∞ < x < ∞) |
| **Expectation**  If an experiment is conducted repeatedly for large number of times under essentially homogeneous conditions, then the average of actual outcomes i.e. the mean value of the probability distribution of random variable is the expected value. Let X be a discrete random variable with PMF p(x) or PDF f(x) then its mathematical expectation is denoted by E(x) and is defined as    **Properties:**    **Variance:** Variance of r.v. X is defined as    Also:  **Properties:**    **Standard Deviation:** |
| **Moments**   1. **The rthmoment of a r.v. X about any point (X=A) is given by** 2. **Ordinary moments /Raw moments (moments about origin):**   The rthraw moment of a r.v. X (i.e. about A= 0 ) is given by     1. **Central moments (moments about mean):**   The rth is given by central moment of a r.v. X (about A = x)     1. **Central Moments in terms of Raw moments**     **Moment Generating Function**:  Suppose X is a discrete random variable, discrete or continuous. The Moment generating function (mgf or MGF) is defined and denoted by :  (for discrete variable)  (for continuous variable)  Also by Taylor series  **Remark:** If the mgf exists for a random variable X, we will be able to obtain all the moments of X. It is very plainly put, one function that generates all the moments of X.  **Result**: Suppose X is a random variable (discrete or continuous) with moment generating function then the rth raw moment is given by |